# Topic 3: Net Present Value

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Corporate Finance – ECONS4280

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In this note, we turn to defining *net* present value (NPV). This concept is key to the field of corporate finance, as it serves as the bedrock of appropriate valuation mechanisms and investment selections. We'll begin by describing what NPV is, then discuss why we use it. We'll also explore alternative investment selection criteria, and use them to show why NPV is the right choice!!

## 1 The Present Value of a Fish

Suppose you are an ancient Briton, carving a life for yourself from the verdant forests and fields of pre-Roman Britain. You lack any formal market place, means of refrigeration or storage, or specialised labor of any kind.

Your goal is to live. To achieve that goal, you need to intake at least as many calories as you expend on average. **The value of a food item to you is therefore its calorie content**.

One food item available to you is fish (no chips yet, sadly). Suppose that a fresh fish, caught today, has 300 calories. A day old fish has no calories —it goes off and becomes *unpalatable*. Suppose you can be assured of catching one fish everyday if you decide to become a **fisherman**. What is the present value of this project, i.e. of becoming a fisherman? Recall the formula:

$$PV(Y) = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t y_t$$

In the example above, the payoffs of a fish if you eat it are $\{y_t^{eat}\} = \{300, 300, 300, ...\}$. The payoffs if you *don't* eat it are $\{y_t^{keep}\} = \{0, 0, 0, ...\}$, so don't do that. Thus, the

Figure 1: Two Ancient Britons, off for a wander in the country!

PV of becoming a fisherman is given by:

$$\begin{aligned} PV(fisherman) &= \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t y_t \\ &= 300 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \\ &= \frac{1+r}{r} 300 \end{aligned}$$

I know what you're thinking: 'Wow! What a neat and not at all unnecessarily formal and complicated way of thinking about food!' To that, all I can say is thank you, I appreciate it :) Continuing with this entirely appropriate thought experiment, let's now consider an alternative food source: a deer.

Deers have many more calories than fish. One speculative source suggests that the total number of calories in a deer are 163,680, but that's if you eat everything. Suppose instead that you can only get 2,000 calories out of each deer you catch. Again, as we don't have any storage, you have to eat all those calories today, or the deer meat will spoil —it will be a big meal. Suppose you can guarantee you will catch a deer once every 10 days, i.e. whenever $t$ is a multiple of 10, including $t = 0$.

What's the PV of becoming a hunter?

$$
\begin{aligned}
PV(hunter) &= \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^{t} y_t \\
&= \left(\frac{1}{1+r}\right)^{0} 2,000 + \left(\frac{1}{1+r}\right)^{10} 2,000 + \left(\frac{1}{1+r}\right)^{20} 2,000 + ... \\
&= 163,680 \sum_{t=0}^{\infty}\left(\left(\frac{1}{1+r}\right)^{10}\right)^{t} \\
&= \frac{(1+r)^{10}}{(1+r)^{10}-1} 2,000
\end{aligned}
$$

Which is better: being a fisherman or being a hunter? Well, it depends! On what? On the value of $r$. Figure 2 demonstrates: if the value of $r$ is less than around 0.1, then being a fisherman produces more present value. If $r$ is greater than that, then the PV of being a hunter exceeds that of being a fisherman.

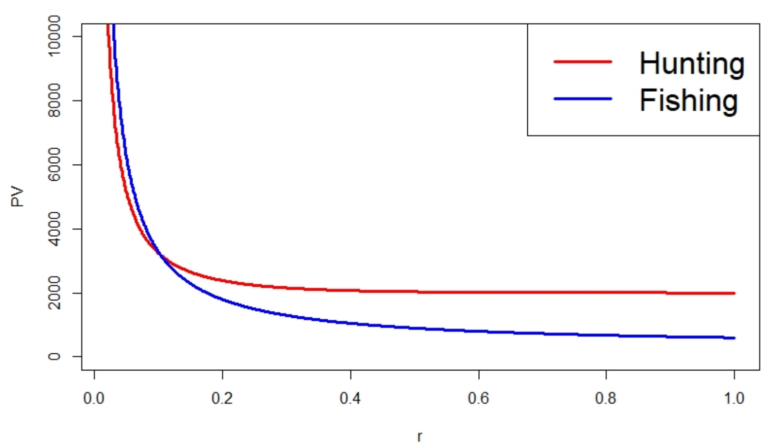



Figure 2: Comparison of the PV of being a Fisherman or Hunter—depending on the discount rate, $r$, either activity could deliver more present value

It's hard to think about what might determine the value of $r$ in pre-Roman Britain. Nonetheless, this example illustrates again that the value of $r$ plays a large role in determining not just the PV of a project, but the **relative** PV too.

### 1.1 'A Grand don't come for free'[1]

What did I say above? You care about ensuring you intake *at least as many* calories as you *expend*. Where have we considered calorie expense? We haven't! Fish do not fall from the sky, deers do not collapse at our sight. Indeed, to catch a fish we might even need something like a **net??!!!???** (GET IT???????)

Projects never deliver returns for free (here's looking at you Terra!); we always have to outlay some expense in order to access payoffs. It is the *balancing* of those outlays alongside the associated payoffs that summarises the allocative decision we face. With that in mind, here's a new definition:

**Definition.** *The* ***Net Present Value*** *of a stream of payments,* $Y = \{y_0, y_1, y_2, y_3, ...\}$ *is equal to the* ***sum*** *of the* ***discounted*** *values of those payments, where the* ***discount rate****,* $r$*, accounts for the* ***opportunity cost*** *of not being able to invest the money and earn a return,* ***minus*** *the discounted costs associated with the project,* $X = \{x_0, x_1, x_2, x_3, ..\}$*:*

$$\begin{aligned} NPV(Y) &= PV(Y) - PV(X) \\ &= \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t y_t - \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t x_t \end{aligned}$$

Fishing is easy work. Hunting, not so much. It seems plausible that our Ancient Britons expend more calories in search of deer than they do sat gamely discussing politics while dipping for fish in the River Thames! Until we include these considerations in our calculus, it's hard for us to give good advice to these people on whether they should be fishermen or hunters. And don't worry! You will get the chance to do just that! In the homework :)

## 2 Net Present Value

The Net Present Value of a project captures the *profit* that accrues to the execution of that project. It encompasses both the benefits and the costs associated with the project. Projects with positive NPV are profitable, and so are viable investments

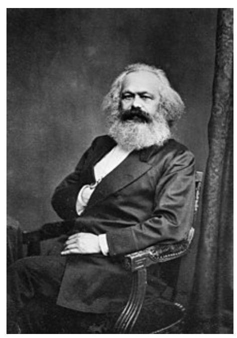
[1] This is a reference to an *iconic* British/English album by the group 'The Streets'. They are an incredible group of musicians, fronted by the inimitable Mike Skinner, who offer some of the most insightful and biting commentaries on English working class life that you'll find in music. Very much worth a listen, if you haven't heard it before BUT: be warned, it is *very* English.

for managers to pursue. This brings us to our definition of the NPV rule of project selection:

**Definition.** *The **NPV Rule** of investment selection is to accept a profit if the NPV is greater than 0. If given a choice between projects, and only one can be chosen, then the investment with the highest NPV should be chosen.*

The NPV rule is the gold standard for project evaluation. It guarantees that projects that maximise shareholder value are the projects chosen for implementation. We'll get to why this is the case in a second, but first a brief philosophical digression...

## 2.1 Marx's views on 'Surplus Value'



Before you panic at my invoking of Karl Marx, rest assured I am not a Marxist! That said, Marx is undoubtedly the most influential economist of all time, so we ought to take seriously his comments on how capitalism works, even if we disagree with them.

In 'Das Kapital', Marx critiques the notion of 'surplus value'. He defines this concept as the difference between the amount raised through a sale of a product and the amount it cost to the owner of that product to manufacture it. Marx describes this surplus value as an extraction of wealth from workers to capitalists, who profit from the labor of workers *whilst inputting no labor themselves*. Capturing this 'surplus value' is central to his arguments in favor of communism, as he views this surplus value as an inefficiency that privileges the wealthy over the proletariat.

Notice any similarities? In many ways, surplus value mirrors a positive NPV project. Simply by funding a project, managers are able to accrue *more returns than they pay in costs*. So, by having access to capital, capitalists are able to generate more capital. Should we agree with Marx? Is this extraction?

Your answer to that question will depend to a great degree on which area of the political compass you land on. One point I would make is that Marx does not consider 'entrepreneurial effort', and the process of allocation, as being meaningful labor. Under this belief, Marx's view may be appealing. But I don't agree that project selection, evaluation, and execution does not involve effort and difficulties on the part of the capitalist. As we mentioned in our first class, finding positive NPV projects is not easy, and often people fail to achieve profitable investments. It is also worth noting that at the time that Marx wrote 'Das Kapital' (1867-83), the state of industrial production was very different to its modern form. Marx's narrative of a gentleman class of casual industrialists does indicate an exploitative relationship between capitalist and worker, especially when placed in the context of Western Imperialism in the 19th Century. Whilst these exploitative relationships no doubt continue to exist, the situation compared to 100-150 years ago is quite different.

Anyway, I think we ought to see positive NPV projects as representing the returns to entrepreneurial effort, the cost and importance of which has grown as the economy has become more open and democratic.

## 2.2 Why use NPV

What is the goal of project selection? As we mentioned in the first class, the goal is to **maximise the returns to shareholders**. We want to pick the projects that deliver the best value to shareholders, because that is the raison d'etre of a firm. NPV is the workhorse of project evaluation, it is used everywhere by almost everyone. This is because it is the *only* project evaluation technique that guarantees that projects will be picked according to shareholder value maximisation. It is also a simple and concise method for collapsing a stream of payments into a single number, which makes it very easy to compare projects. But why does NPV maximise *shareholder value*?

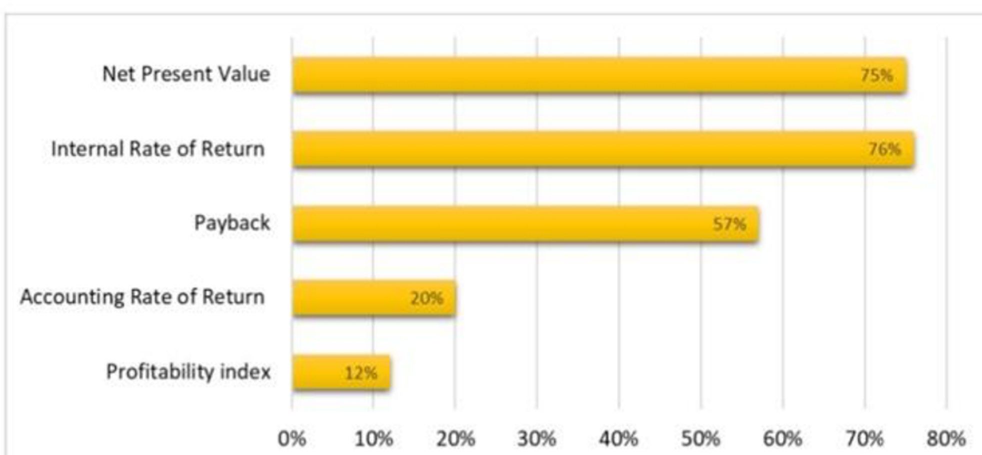
The key feature of NPV is the use of a discount rate designed to capture the opportunity cost of the project. By discounting future payoffs by the opportunities lost *by the shareholders*, the value of the project is *directly* comparable to the alternative investment strategies of shareholders. If a project has a positive NPV, it means it is delivering value to shareholders. If project $A$ has the highest NPV out of all available projects, $\{A, B, C, ..., Z\}$, then it is delivering *the most value* to shareholders of all projects.

Of course, the key nuance/assumption we need to land to make this claim is that the selection of the discount rate matches the opportunity costs of investors. This is no easy task... Opportunities have two components: risk and reward. When we're estimating the value of *our* project, we want to compare it to alternative opportunities that have *similar levels of risk and reward*. Then we can say, 'well the interest rate on those opportunities is roughly $r$, so we should use that rate to discount this project'. Thus, what NPV is doing is comparing a project *relative to other similarly risky/rewardy projects*.

Right now though, we're in a world without risk. We'll think more about how we select the discount rate, including how we measure risk, when we move onto the next Super Topic.

## 2.3 Alternative Investment Selection Criteria

Take a look at Figure 3:




Source: J. R. Graham and C. R. Harvey, "The Theory and Practice of Finance: Evidence from the Field," *Journal of Financial Economics* 61 (2001), pp. 187-243.

Figure 3

This figure reports survey evidence on the percentage of CFOs who always, or almost always, use a particular technique for evaluating investment projects. Note that whilst NPV is right at the top (though slightly outperformed by the 'Internal Rate of Return' metric), there are many alternative methods also used frequently my CFOs. Why might that be? How appropriate are these alternatives, and do they challenge the supremacy of NPV as outlined above?

To illustrate more explicitly why NPV is the right choice of methodology for investment selection, we're going to look at three of the alternatives outlined in Figure 3, and identify where and why these criteria might fail to encourage project selection that maximises shareholder value. They are:

1. Payback period rule.
2. Book rate of return.
3. Internal rate of return (IRR).

We'll spend most time talking about IRR. The reason for this is twofold: firstly, as Figure 3 shows, it is an extremely popular technique, so we ought to understand how it works. Secondly, if implemented correctly, it is an equally appropriate selection criteria to the NPV rule. In fact, the IRR rule will produce the same ordering of projects as the NPV rule.

### 2.3.1 Payback period rule

The payback period rule is an investment selection criteria that emphasises the time it takes for the initial outlay to be recovered. Put simply, this criteria tells managers to select projects that *pay back the fastest*.

Like NPV, this criteria allows us to compare projects based on a single number: the payback period. As a consequence, comparison of projects is also simple. However, under the payback period rule, managers can be incentivised to select projects *that do not maximise shareholder value*. To illustrate with an example, consider the following three projects, that payout in three periods, with each requiring an initial outlay of $2,000:

| **Project** | **Outlay** | $Y_1$ | $Y_2$ | $Y_3$ | **Payback Period** |
|---|---|---|---|---|---|
| A | -2,000 | 500 | 500 | 5,000 | 3 |
| B | -2,000 | 500 | 1800 | 0 | 2 |
| C | -2,000 | 1800 | 500 | 0 | 3 |

Under the payback period rule, we should be indifferent between projects B and C, and we should prefer either to Project A. There are two issues here:

- The NPV of Project C is strictly higher than Project B for any $r > 0$
- The value of $r$ that would make Project A worth less than Project C would need to be extremely high.

The first point should be obvious: in Project C *we get money now*. For the second point, consider the following:

$$NPV(A) = 500 + \frac{1}{1+r}500 + \frac{1}{(1+r)^2}5,000$$

$$NPV(C) = 1,800 + \frac{1}{1+r}500$$

If $NPV(C) > NPV(A)$, then:

$$\begin{aligned}
1,800 + \frac{1}{1+r}500 &> 500 + \frac{1}{1+r}500 + \frac{1}{(1+r)^2}5,000 \\
1,800 &> 500 + \frac{1}{(1+r)^2}5,000 \\
1,300 &> \frac{1}{(1+r)^2}5,000 \\
(1+r)^2 &> \frac{5,000}{1,300} \\
r &> \left(\frac{5,000}{1,300}\right)^{\frac{1}{2}} - 1 \\
r &> 0.96116...
\end{aligned}$$

So the opportunity cost of taking on these projects would need to deliver a per-period return of roughly **96%** for the NPV of Project C to exceed that of Project A. There are almost no opportunities that deliver returns of this kind (the average equity premium is roughly 8%).

**In summary:** the payback period rule does not incorporate the notion that money has *time value*: Project B and Project C look equally good under the period payback rule. It also entirely ignores payoffs that occur after the cutoff, irrespective of how large they can be. This can result in selections that do not maximise shareholder returns, and induces an excessively *short-term* attitude towards projects.

**So, why might managers want to follow this rule?** There are many incentives that managers face to prioritise *short term returns*. CEOs are often fired for missing quarterly/annual earnings forecasts, teams lose funding for poor quarterly results, and people are desperate for promotions. All of these incentives could encourage managers to prioritise deals that pay off *quickly* rather than *effectively*.

## 2.4 Book rate of return

The book rate of return selection criteria says that projects should be picked according to the *ratio of book income to book assets* that the project will effect. That is, how will the inflows affect **book** income, relative to how will outflows affect **book** assets. Typically the rule suggests that projects that *equal or exceed* the average of past ratios should be pursued, and those that subceed should be jettisoned.

$$\text{Book rate of return} = \frac{\text{book income}}{\text{book assets}}$$

What do we mean by *book*? Accountants do not earn big bucks for nothing! The *book* value of income or asset investment flows from a project will often differ from the *actual* cash flows involved in those activities. For example, an accountant might label some outflows as **capital investments**, whilst others would be considered **operating expenses**. Take the construction of a new power station: some of the costs will be investments in fixed capital (cooling towers, furnaces, storage), whereas other expenses will maintain operation of the plant (employees, repairs, inputs). Operating expenses hit book income immediately, *but capital expenditures do not*. Instead, the capital becomes an *asset* on the books balance sheet, and is depreciated over time. Only the depreciation is deducted from book income each year.

By focusing on the *book income*, and changes in *book assets*, the book rate of return does not necessarily capture the actual cash flows associated with the project. The merits of a project ought not to depend on *how accountants categorise cash flows*[2].

**So, why might managers want to follow this rule?** For similar reasons as the 'payback period rule': shareholders tend to pay more attention to book measures of profitability, rather than cash flows. Managers face short-term pressures to meet shareholders' expectations of profitability, and so may be tempted to focus more on book returns rather than actual returns.

[2]Once we take into account tax consequences of course! We'll get to that next class

## 2.5 Internal Rate of Return

As Figure 3 shows, CFOs very often make use of the internal rate of return to conduct project evaluation. As such, we should treat this alternative with a commensurate degree of attention. So, what is the **internal rate of return**?

**Definition.** *The **internal rate of return** (IRR) is the rate of discount that makes the NPV of a project equal to 0. So, for a project lasting $T$ years, with inflows $\{y_t\}$, and outflows $\{x_t\}$, the IRR is the solution to the following expression:*

$$NPV = (y_0 - x_0) + \frac{1}{1+IRR}(y_1 - x_1) + \frac{1}{(1+IRR)^2}(y_2 - x_2) + ... + \frac{1}{(1+IRR)^T}(y_T - x_T) = 0$$

Does this seem familiar? There is a clear similarity between this methodology, and the 'yield-to-maturity' that we saw in a previous class! The intuition is much the same: 'what value must I discount net flows by so as to render the return of the project equal to zero?' Note that in the same way as the yield, *the internal rate of return is also the average rate of return of the project.* The challenges of solving for the IRR are also the same as in the 'yield-to-maturity' case —solving polynomials is not easy! Let's consider an example to illustrate how we might go about doing it.

### 2.5.1 An example IRR exercise

Suppose we are considering a project that has the following inflows and outflows:

| | *2022* | *2023* | *2024* | *2025* | *2026* |
|---|---|---|---|---|---|
| Inflows | \$0 | \$1,500 | \$1,500 | \$2,750 | \$0 |
| Outflows | -\$2,000 | -\$500 | -\$500 | -\$500 | -\$1,000 |
| | | | | | |
| Net | -\$2,000 | \$1000 | \$1000 | \$2,250 | -\$1,000 |

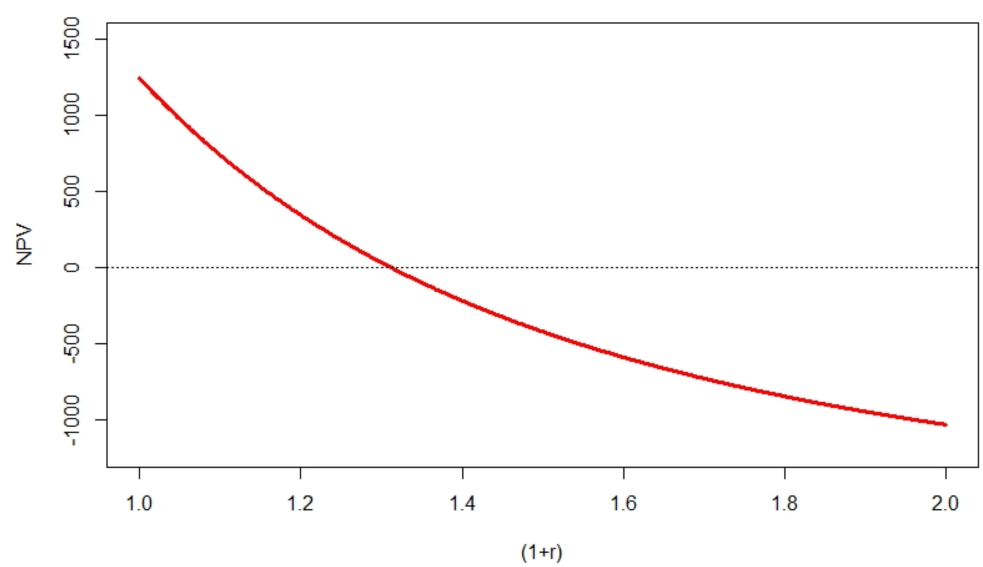
We can imagine that the \$2,000 initial outlay is some capital expenditure, whilst the \$500 are operating expenses. By 2026, the project is no longer producing, and the \$1,000 outflow represents the dismantling costs. The NPV of this project, for a given discount rate $r$, is given by the following expression:

$$NPV = -2,000 + \frac{1}{1+r}1,000 + \frac{1}{(1+r)^2}1,000 + \frac{1}{(1+r)^3}2,250 - \frac{1}{(1+r)^4}1,000$$

Now to calculate the NPV, we would find the return on a similar opportunity to the project in question, and use this return as our discount rate, $r$, and this would tell us whether the project had positive, neutral, or negative NPV. To calculate the IRR though, we instead find the value of $r$ such that:

$$-2,000 + \frac{1}{1+r}1,000 + \frac{1}{(1+r)^2}1,000 + \frac{1}{(1+r)^3}2,250 - \frac{1}{(1+r)^4}1,000 = 0$$

What we're looking for is the *roots* of this polynomial. In other words, where does the function defined by this line meet the origin? As we discussed previously, no general solution for finding the roots of a polynomial of degree four exists, so as before, we have to solve this numerically. We can see the solution clearly if we graph the expression above, using $(1+r)$ as our x-axis, and the NPV as the y-axis. Note that we'll restrict to positive values of $r$, as a negative opportunity cost is not a sensible concept[3]. So we know that the value of $(1+r)$ that sets the NPV equal




to zero is going to be around 1.3. We also know that as we increase $r$, the value of the NPV should *decrease*. So, how do we actually find it in practice?

Many computer programs exist that will numerically solve the expression above for you; for example, WolframAlpha is a free software program available via the internet, that will easily find the value of $r$ that sets the NPV equal to zero. Nonetheless,

[3]Remember that you can always hold onto your money and earn $r = 0$

it's perhaps worth offering at least one explicit method you could use to find the value yourself. We'll do that now by using an 'interpolation' approach. That will work something like this:

1. Guess a value for the IRR, say $IRR = 0.3$.
2. Calculate the NPV using this guess for the IRR as the discount rate.
3. If the NPV is positive, increase our guess to $IRR = 0.35$. If the NPV is negative, decrease our guess to $r = 0.25$[4].
4. Calculate the NPV again.
5. If the NPV goes from positive to negative, or vice versa, we know that the true value of the IRR is in this range.
6. Take the midpoint of that range, and repeat until you reach a sufficient degree of precision.

Let's try that out. Suppose we guess that $IRR = 0.3$. Then:

$$NPV = -2,000 + \frac{1}{1.3}1,000 + \frac{1}{(1.3)^2}1,000 + \frac{1}{(1.3)^3}2,250 - \frac{1}{(1.3)^4}1,000 \approx \$34$$

So, the true IRR must be slightly above 0.3. Let's try 0.35:

$$NPV = -2,000 + \frac{1}{1.35}1,000 + \frac{1}{(1.35)^2}1,000 + \frac{1}{(1.35)^3}2,250 - \frac{1}{(1.35)^4}1,000 \approx -\$98$$

Ah! So the true IRR must be **below** 0.35. Let's try 0.325:

$$NPV = -2,000 + \frac{1}{1.325}1,000 + \frac{1}{(1.325)^2}1,000 + \frac{1}{(1.325)^3}2,250 - \frac{1}{(1.325)^4}1,000 \approx -\$33$$

Now we know the value must be between 0.3 and 0.325. Let's try 0.3125:

$$NPV = -2,000 + \frac{1}{1.3125}1,000 + \frac{1}{(1.3125)^2}1,000 + \frac{1}{(1.3125)^3}2,250 - \frac{1}{(1.3125)^4}1,000 \approx \$0.57$$

So we are very close! If we keep doing this, we will eventually arrive at:

$$IRR = 0.31271$$

[4]Why? Because we know that as we increase $r$, the value of the NPV should *decrease*.

Let's compare this to the output from WolframAlpha. First, what do I put into WolframAlpha to get it to solve my problem:

WolframAlpha computational intelligence.

find x such that -2000+1000*((1+x)^(-1))+1000*((1+x)^(-2))+2250*((1+x)^(-3))-1000*((1+x)^(-4)) =0

After pressing enter, what do I find?

Input interpretation

solve $-2000 + \frac{1000}{1+x} + \frac{1000}{(1+x)^2} + \frac{2250}{(1+x)^3} - \frac{1000}{(1+x)^4} = 0$ for $x$

Results    More digits    Exact forms    Step-by-step solution

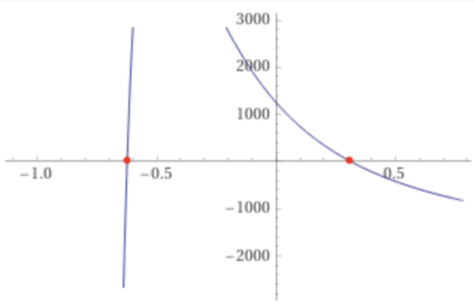
$x \approx -0.62419$

$x \approx 0.31271$

$x \approx -1.5943 - 0.8126\,i$

$x \approx -1.5943 + 0.8126\,i$

Root plot




There are four solutions to our polynomial: two are 'real', and two are 'complex' numbers. The notion of a 'complex' opportunity cost is hard to conceptualise, so let's throw these out. Of the two remaining real roots, only one is positive, and it's the value we found using our method above. So, that's how we find the IRR: we write our project in NPV form for a generic $r$, then find the $r$ that sets the NPV equal to 0.

### 2.5.2 The IRR Rule

Once we have established the IRR, how do we implement it in project selection?

**Definition.** *The* ***internal rate of return rule*** *is to accept an investment project if the opportunity cost of capital is* <u>*less*</u> *than the internal rate of return.*

Why does this make sense as a rule? Well, consider our worked example above: if the IRR is greater than the opportunity cost of capital, then it follows that the NPV of the project *must be positive*. Similarly, if the IRR is *less* than the opportunity cost of capital, then the NPV must be *negative.* Thus, when our NPV is a decreasing function of the discount rate $r$, *the IRR rule and the NPV rule are equivalent*. But what about when the NPV of a project is *not* decreasing in the discount rate...?

### 2.5.3 Where the IRR Rule fails

There are four pitfalls to be aware of when applying the IRR rule. They are:

1. Lending vs. borrowing
2. Multiple rates of return
3. Mutually exclusive projects
4. Multiple opportunity costs of capital

Lets look at each in turn.

### 2.5.4 IRR Pitfalls: (i) Lending vs. borrowing:

Suppose that we're comparing two projects, one in which we are borrowing money, and the other in which we are lending money. We could think of this as the choice between issuing or buying a bond, for example. Suppose the two projects have the following net cash flows:

| | *2022* | *2023* |
|---|---|---|
| A | \$2,000 | -\$2,500 |
| B | -\$2,000 | \$2,500 |

The NPV of these two projects is:

$$NPV(A) = 2,000 - \frac{1}{1+r}2,500$$

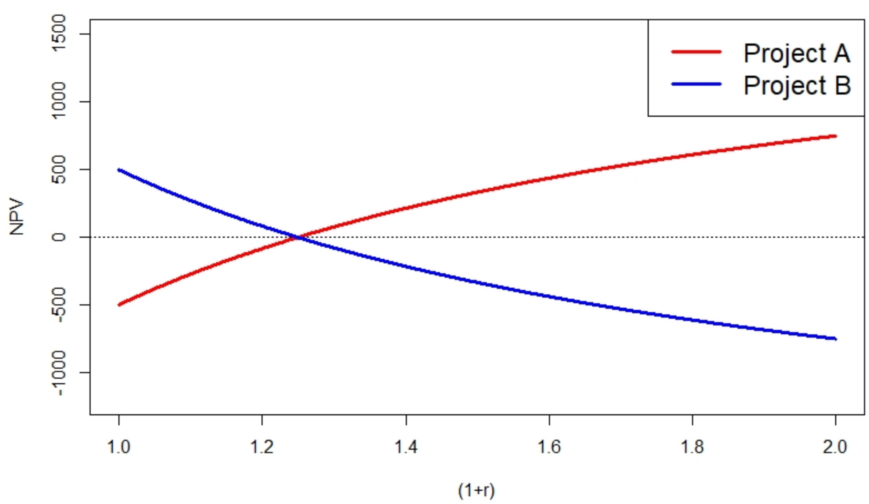
$$NPV(B) = -2,000 + \frac{1}{1+r}2,500$$

The IRR can be found easily for these simple projects:

$$\begin{aligned} 2,000 - \frac{1}{1+IRR^A}2,500 &= 0 \\ \therefore 2,000 &= \frac{1}{1+IRR^A}2,500 \\ \implies 2,000(1+IRR^A) &= 2,500 \\ \therefore IRR^A &= \frac{2,500}{2,000} - 1 = 0.25 \end{aligned}$$

$$\begin{aligned} 2,000 - \frac{1}{1+IRR^B}2,500 &= 0 \\ \therefore 2,000 &= \frac{1}{1+IRR^B}2,500 \\ \implies 2,000(1+IRR^B) &= 2,500 \\ \therefore IRR^B &= \frac{2,500}{2,000} - 1 = 0.25 \end{aligned}$$

So, $IRR^A = IRR^B$. But what about the NPV? Let's plot both using $1+r$ as the x-axis: Only when the discount rate is exactly equal to the IRR (i.e. $r = 0.25$), is




the NPV of these two projects the same. If the opportunity cost of capital is *less* than 0.25, then the IRR rule would tell us to accept **both** projects. However, we can clearly see that for any value of $r$ in this range, *the NPV of Project A is negative* —it is **not** a profitable project.

Unlike Project A, the NPV of Project B is **increasing** in the discount rate, $r$. In this scenario, the IRR rule gives us *exactly the wrong answer*. The reason why the NPV is increasing in the discount rate is because this project involves *borrowing* rather than *lending*. In these scenarios, the IRR rule does not maximise shareholder value; in fact, it does the opposite.

### 2.5.5 IRR Pitfalls: (ii) Multiple rates of return

Occasionally, the payoffs of a project will be such that there is *not just one* valid IRR. Consider a modification to the power plant project we discussed previously. Now, the plant is more profitable as it operates, but the clean-up cost is much greater: We can calculate the NPV as before:

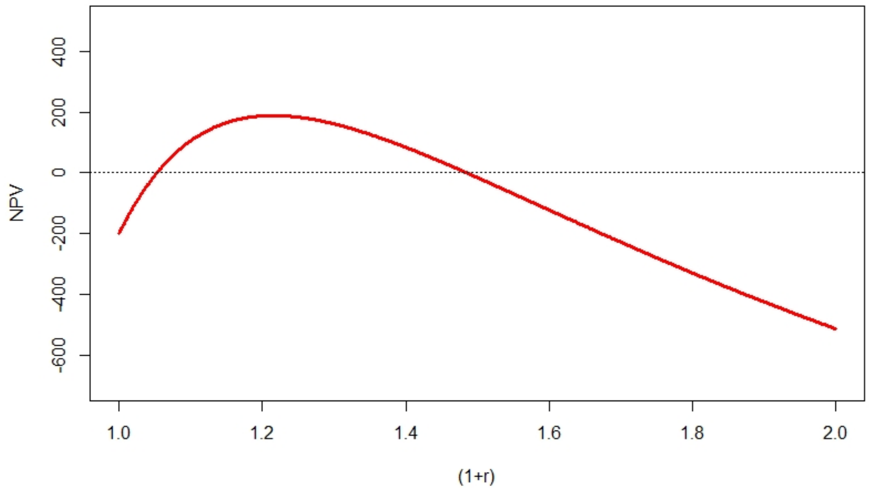
| | *2022* | *2023* | *2024* | *2025* | *2026* |
|---|---|---|---|---|---|
| Inflows | $0 | $2,500 | $2,500 | $2,500 | $0 |
| Outflows | -$2,000 | -$500 | -$500 | -$500 | -$4,200 |
| | | | | | |
| Net | -$2,000 | $2,000 | $2,000 | $2,000 | -$4,200 |

$$NPV = -2,000 + \frac{1}{1+r}2,000 + \frac{1}{(1+r)^2}2,000 + \frac{1}{(1+r)^3}2,000 - \frac{1}{(1+r)^4}4,200$$

We set it to zero and find the IRR:

$$-2,000+\frac{1}{1+IRR}2,000+\frac{1}{(1+IRR)^2}2,000+\frac{1}{(1+IRR)^3}2,000-\frac{1}{(1+IRR)^4}4,200 = 0$$

We can plot the function of the NPV as a function of $(1 + r)$ again:




Now we have a problem! We have two values for the IRR. They are, roughly, $IRR_1 = 0.053$, and $IRR_2 = 0.485$. Which do we pick? Well, if we pick $IRR_1$, then we can see that the IRR rule will fail to select profitable projects: for any opportunity cost *below* $IRR_1$, the NPV of the project is *negative*. Even selecting $IRR_2$ will not solve the problem: if $IRR_2$ is our threshold, the IRR rule would tell us to accept a project if the opportunity cost were, say, $r = 0.05$. But we can clearly see that this project would deliver *negative profit*. Again, the issue here is that, for some regions of the NPV function, *the NPV is **increasing** in the discount rate $r$*.

### 2.5.6 IRR Pitfalls: (iii) Mutually Exclusive Projects

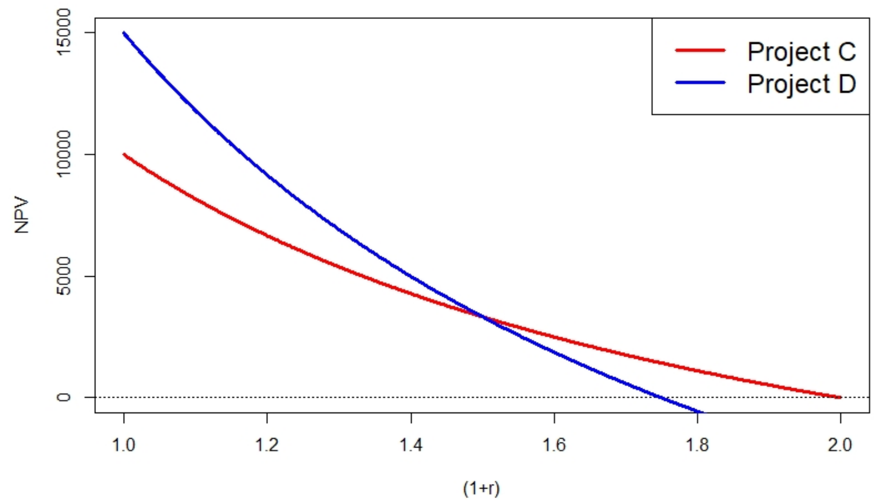
When we looked at NPV, we said that, if we have to choose only one project out of many, we should pick the project that delivers the highest NPV. Is that true for the IRR rule? Consider two projects: So the NPVs of these projects are:

| | *2022* | *2023* |
|---|---|---|
| C | -$10,000 | $20,000 |
| D | -$20,000 | $35,000 |

$$NPV(C) = -10,000 + \frac{1}{1+r}20,000$$

$$NPV(D) = -20,000 + \frac{1}{1+r}35,000$$

The value of $r$ that sets these two NPVs equal to zero is 100% and 75% respectively. Does this mean we should pick Project C over Project D? What about their NPVs? This figure clearly shows that, for most reasonable expectations of the opportunity




cost of capital (i.e. less that around 50%), Project D should be preferred. Thus, the IRR does not rank projects according to their profitability without further modification. We can resolve this issue with IRR by looking at *incremental flows*: So

| | *2022* | *2023* | *IRR* | *NPV at 10%* |
|---|---|---|---|---|
| D-C | -\$10,000 | \$15,000 | 50 | \$3,636 |

once we look at *incremental* cash flows, then we can see that it is worth the extra investment to pursue Project D. But a natural question here is: *why bother?* We don't need to do this extra step; we could just rely on NPV.

### 2.5.7 IRR Pitfalls: (iv) Multiple costs of capital

Suppose that, as in the case with bonds, we imagine that there are multiple opportunity costs of capital across time:

$$NPV = y_0 + \frac{1}{1+r_1}y_1 + \frac{1}{(1+r_2)^2}y_2 + ...$$

The IRR rule tells us to accept projects where the opportunity cost of capital is less than the internal rate of return. In this case, which opportunity cost of capital do we choose as our threshold determinant? It is really not clear.

### 2.5.8 So, why are people using IRR?

A good question! There is no theoretical justification for implementing IRR over NPV. As we have seen, it can order rankings in the same way as NPV, but it can just as equally fail to do so. There are no equivalent pitfalls associated with the NPV approach. *So, the choice to select IRR must be **behavioral**.* There are at least two reasons why managers might prefer IRR over NPV as a project selection rule:

- Rates are *easier to understand* than aggregates.
- IRR overvalues projects with *short term returns*.

In the case of the former, this should be clear: understanding that a project pays $X\%$ return a year is intuitive, whereas understanding that a project pays $\$Y$ is harder to conceptualise in terms of value. In terms of the later, consider the following two projects:

| | *2022* | *2023* | *2024* | *2025* | ... |
|---|---|---|---|---|---|
| E | -\$10,000 | \$7,000 | \$6,500 | \$4,500 | \$0 |
| F | -\$10,000 | \$2,000 | \$2,000 | \$2,000 | \$2,000 |

In Project E, we outlay \$10,000 for a project that pays out for three years, then ceases to be productive. In Project F, we outlay the same amount, \$10,000, for a *perpetuity* that pays out \$2,000 forever. What is the IRR of these two projects? First the NPVs:

$$NPV(E) = -10,000 + \frac{1}{(1+r)}7,000 + \frac{1}{(1+r)^2}6,500 + \frac{1}{(1+r)^3}4,500$$

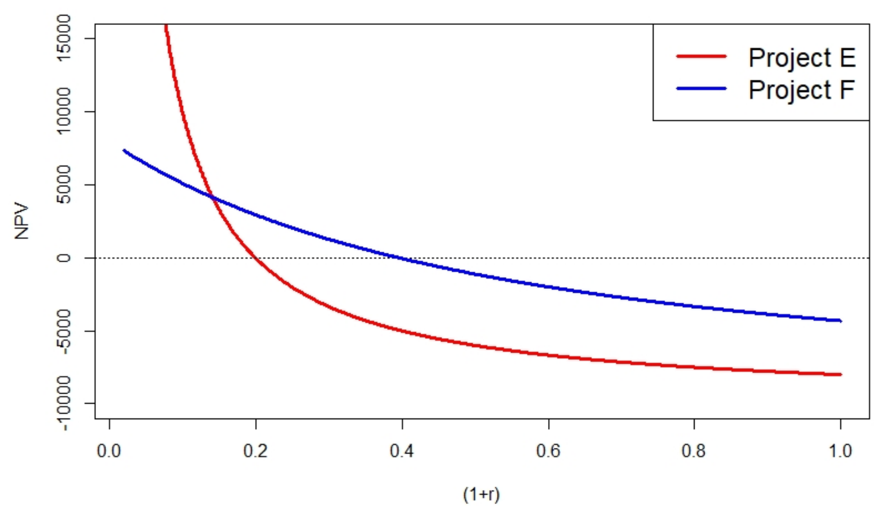
$$NPV(F) = -10,000 + \sum_{t=1}^{\infty} \frac{1}{(1+r)^t}2,000$$

The IRR implied by Project E, found by plugging into WolframAlpha, is $IRR^E \approx$

0.40. The IRR implied by Project F can be solved analytically:

$$\begin{aligned}
10,000 &= \sum_{t=1}^{\infty} \frac{1}{(1+IRR^f)^t} 2,000 \\
&= \frac{1}{(1+IRR^f)} \sum_{t=0}^{\infty} \frac{1}{(1+IRR^f)^t} 2,000 \\
&= \frac{1}{(1+IRR^f)} \frac{(1+IRR^f)}{IRR^f} 2,000 \\
&= \frac{1}{IRR^f} 2,000 \\
\therefore IRR^f &= 0.2
\end{aligned}$$

So Project F has a lower IRR than Project E. What about their NPVs for a given value of $r$: So for any opportunity cost of capital less than around 14% (fairly




high!), the *long-term* project of $E$ is a better NPV option than that of $F$. However, as we've discussed, *managers face a number of pressures to prioritise short-term returns.* Perhaps it is this pressure that encourages them to implement the IRR rule?