# Topic 6: Valuing Stocks 

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Now that we've talked about present value, the opportunity cost of capital, the notion of uncertainty, and the relationship between risk and reward, we have all the tools we need to talk about the value of stocks. Stocks are very different from bonds, the other financial instrument that we've mentioned previously. Whereas bonds pay certain payouts in the form of coupon payments and principals, stocks pay uncertain payouts: dividends. Similarly, whilst bonds have a fixed maturity, shares never 'mature': you can hold them forever, or at least for as long as the company exists. Most importantly, as an owner of a share you are a part-owner of the company that issued that share-if you own $50 \%$ of the shares, you own $50 \%$ of the company.

We'll start by discussing the difference between 'public' and 'private' firms. We'll then talk about what cash flows you access by owning a stock. We'll then convert those cash flows into Present Value. In particular, we will emphasise that the Present Value ought to be equal to the price, and that the price is simply the sum of the discounted expected future dividends.

## 1 Public vs. Private Firms

Suppose you go into Robinhood and decide you'd like to buy a share in a company. Suppose for now that the company is Apple. Well, no problem! You look up Apple, see how much the price is, make sure you have enough cash in your Robinhood account, and make the buy. Easy!

What if you had your heart set on a different company. As a Snickers obsessive, you want to express your heartfelt love and respect for the fine company that pro-
duces this magnificent chocolate bar: Mars. Can you buy a share in them? Let's open up Robinhood... no!! They're not listed for sale! What's going on here?

The difference is that Apple is a publicly traded company, whereas Mars is a private company. At Apple, shares in the company are periodically created, issued, and made available to purchase by anyone in the public with enough money to buy them. People then openly trade these shares between themselves via stock exchanges like the NYSE, LSE, and SEHK. By contrast, Mars issues no such public shares. Mars is the private possession of its shareholders, and they cannot list their shares on any stock exchange. If someone wishes to buy shares in Mars, they must engage in a private and direct interaction with someone who holds shares in Mars.

## 2 Dividends, dividends, dividends,...

In this section we will derive the present value of a stock. We will show that the present value of a stock is entirely a function of expected dividends going into the future. We begin by clarifying the 'return' on a stock. We then show that with manipulation, we can convert that return into an expression for present value, where the price of the stock at the point of purchase simply reflects the present value of future discounted dividends.

### 2.1 What is a stock 'return'

We have mentioned stock 'returns' a lot. What does that mean? Well, let's consider the annual return of buying a stock at price $P_{0}$. At the end of the year, what will ownership of that stock provided to me?

- Dividend payment, $D I V_{1}$
- Capital gain/loss, $P_{1}-P_{0}$

Thus, the return on an asset is simply given by the ratio of these provisions over the price that I paid:

$$
r=\frac{D I V_{1}+P_{1}-P_{0}}{P_{0}}
$$

Note that there is both an ex-ante and ex-post return: the ex-ante return is what I expect to receive, and the ex-post is what I actually receive. This is because both the dividend payment and the future price are uncertain.

### 2.2 What determines dividends?

A good question! The decision to issue dividends is made by the board, which is made up of both executive and non-executive directors. There are many reasons why boards would or would not issue dividends, and also how large those dividends will be - it is an open question in the field.

However, in general, the following two statements are good rules of thumb:

- Companies looking to grow rapidly will typically not issue dividends, or issue limited dividends, so that profits can be reinvested.
- Companies in stable environments will issue larger and more regular dividends to attract investment.

To illustrate with a real world example: the average dividend yield (the amount of money a company pays shareholders for owning a share of its stock in a year divided by its current stock price) has been between 2 and $5 \%$ for the S\&P 500. Amazon has a dividend yield of $0 \%$ : it has never paid dividends. By contrast, Altria, a major cigarette manufacturer, had a dividend yield of $8.57 \%$. Hopefully you can see how these two companies fit into the two categories above.

The key takeaway here is that future dividends are uncertain at the point of purchase. Ex-post dividends are observable, and may provide an indicator of what to expect for future dividends, but ex-ante dividends are unknown.

### 2.3 From stock returns to present value

Suppose we focus on ex-ante returns, i.e. expected returns. This makes sense if we're thinking about the decision to buy a stock - we can't use ex-post numbers in this case, as the future has not yet happened. Thus, the information that we have is the price today, $P_{0}$, whereas the future price, $P_{1}$, the future dividend, $D I V_{1}$, and the actual return, $r$, are all unknown. Then, re-arranging the equation above:

$$
\begin{aligned}
\mathbb{E}_{0}[r] & =\frac{\mathbb{E}_{0}\left[D I V_{1}\right]+\mathbb{E}_{0}\left[P_{1}\right]-P_{0}}{P_{0}} \\
\mathbb{E}_{0}[r] P_{0} & =\mathbb{E}_{0}\left[D I V_{1}\right]+\mathbb{E}_{0}\left[P_{1}\right]-P_{0} \\
\left(1+\mathbb{E}_{0}[r]\right) P_{0} & =\mathbb{E}_{0}\left[D I V_{1}\right]+\mathbb{E}_{0}\left[P_{1}\right] \\
P_{0} & =\frac{1}{1+\mathbb{E}_{0}[r]} \mathbb{E}_{0}\left[D I V_{1}\right]+\frac{1}{1+\mathbb{E}_{0}[r]} \mathbb{E}_{0}\left[P_{1}\right]
\end{aligned}
$$

So, the price at time 0 , is the expected dividend plus the expected price, both discounted by our expected return. Sound familiar?? This sounds an awful lot like present value. The dividends are the in-flows associated with the stock, and we're discounting them by our expected returns from the stock, i.e. by what we would expect to make from a similar stock. The presence of the price $P_{1}$ would represent the value to us if we sold the stock and realised our capital gains, and this too is discounted by the expected returns/opportunity cost of capital.

We could leave our expression as above. However, as we'll now see, if we substitute away $P_{1}$ (akin to the notion of holding on to the asset for another period), then a startling pattern emerges. First, let's drop the notation of including the $\mathbb{E}_{0}[$.$] and$ just bear in mind that all of these future variables are expected values. Then:

$$
\begin{aligned}
P_{0} & =\frac{1}{1+r} D I V_{1}+\frac{1}{1+r} P_{1} \\
\therefore P_{1} & =\frac{1}{1+r} D I V_{2}+\frac{1}{1+r} P_{2} \\
\Longrightarrow P_{0} & =\frac{1}{1+r} D I V_{1}+\frac{1}{1+r}\left(\frac{1}{1+r} D I V_{2}+\frac{1}{1+r} P_{2}\right) \\
& =\frac{1}{1+r} D I V_{1}+\left(\frac{1}{1+r}\right)^{2} D I V_{2}+\left(\frac{1}{1+r}\right)^{2} P_{2}
\end{aligned}
$$

Now if we substitute away for $P_{2}$ :

$$
\begin{aligned}
P_{0} & =\frac{1}{1+r} D I V_{1}+\left(\frac{1}{1+r}\right)^{2} D I V_{2}+\left(\frac{1}{1+r}\right)^{2} P_{2} \\
P_{2} & =\frac{1}{1+r} D I V_{3}+\frac{1}{1+r} P_{3} \\
\Longrightarrow P_{0} & =\frac{1}{1+r} D I V_{1}+\left(\frac{1}{1+r}\right)^{2} D I V_{2}+\left(\frac{1}{1+r}\right)^{2}\left(\frac{1}{1+r} D I V_{3}+\frac{1}{1+r} P_{3}\right) \\
& =\frac{1}{1+r} D I V_{1}+\left(\frac{1}{1+r}\right)^{2} D I V_{2}+\left(\frac{1}{1+r}\right)^{3} D I V_{3}+\left(\frac{1}{1+r}\right)^{3} P_{3}
\end{aligned}
$$

We can keep doing this forever. If we were to continue this process of substituting out the price term at the end of our expression, we would find that:

$$
P_{0}=\underbrace{\left[\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t} D I V_{t}\right]}_{\text {Discounted Cash Flows }}+\underbrace{\lim _{t \rightarrow \infty}\left(\frac{1}{1+r}\right)^{t} P_{t}}_{\text {The one that got away! }}
$$

We can represent the price today, $P_{0}$, as the sum of all future discounted dividends from owning the stock, plus some $P_{t}$ in the infinite future that we can't substitute away. I include that term for mathematical rigor, but we can typically safely usually assume that this term is equal to 0 , so we needn't worry about it. Therefore, we can say that the price of a stock bought today, $P_{0}$, reintroducing our expectation terms:

$$
P_{0}=\sum_{t=1}^{\infty}\left(\frac{1}{1+\mathbb{E}_{0}[r]}\right)^{t} \mathbb{E}_{0}\left[D I V_{t}\right]
$$

This is a major result in finance: the price of a stock simply reflects the expected future dividends, scaled by the expected return on the stock. Put in words we're by now familiar with: the price of a stock today simply equals the present value of expected future dividends. We'll think about how to identify the expected return in a later class (note that it will be via the 'similarly' risky idea we've considered already), but let's now think about how we estimate expected future dividends.

### 2.4 Forecasting dividends

Forecasting future dividends is very difficult, especially given our previous discussion on the determinants of dividends: there, we concluded that it is hard to say with confidence what drives the dividend decision. This is a problem if we want to take the 'Rational Expectations' paradigm seriously! Recall that a critical feature of that model was that agents understand the random processes that govern the data. We cannot even say with confidence what the random process is! So how do we move forward?

### 2.4.1 Past growth = future growth

A common approach is to estimate a growth rate of dividends, and use that growth rate, $g$, to estimate future dividends:

$$
\begin{aligned}
\mathbb{E}_{0}\left[D I V_{1}\right] & =(1+g) D I V_{0} \\
\mathbb{E}_{0}\left[D I V_{2}\right] & =(1+g)^{2} D I V_{0} \\
\vdots & \\
\mathbb{E}_{0}\left[D I V_{t}\right] & =(1+g)^{t} D I V_{0}
\end{aligned}
$$

How do we estimate $g$ ? There are many ways we could do it, but one natural method might be to assume that past growth is a good predictor of future growth. So, we go to the data, look at past dividends, and calculate their average growth. Then, we just use this as our guide for how we expect future dividends to grow.

So, is it true that past growth is a good predictor of future growth? This is something we can test with data. I collected data on the dividends of a large number of US stocks from 2008-2022. I first calculated the average dividend growth rate of each of the stocks for the period 2008-2016, then calculated the average dividend growth rate for the period 2016-2022. Does the former map onto the latter? Let $g_{i}^{\text {pre }}$ denote the 2008-16 growth rate of firm $i$, and $g_{i}^{\text {post }}$ the 2016-22 growth rate of firm $i$. Then, I run the following regression:

$$
g_{i}^{p o s t}=\beta_{0}+\beta_{1} g_{i}^{p r e}+\epsilon_{i}
$$

In a perfect world, we would have that beta $a_{1}=1$. This would mean that pre growth rates are roughly the same as post growth rates, i.e. if we know pre growth rates, a good guess of post growth rates is just those pre rates. We'd also hope that the errors would be quite small (i.e. the gaps between the line of best fit and the actual data points would be pretty small). Then we could say that, given a pre growth rate, I could give you an estimate of the post growth rate (coming from $\beta=1$ ) that is probably pretty close to what the growth rate ends up being (coming from small errors). What do we actually find?

| Dependent Variable: | $g_{d}^{\text {post }}$ |
| :--- | :---: |
| Model: | $(1)$ |
| Variables |  |
| (Intercept) | $0.2751^{* * *}$ |
| gre | $(0.0176)$ |
| $g_{d}^{\text {pe }}$ | $0.1221^{* * *}$ |
|  | $(0.0370)$ |
| Fit statistics |  |
| Observations | 2,731 |
| $\mathrm{R}^{2}$ | 0.00397 |
| IID standard-errors in parentheses |  |
| Signif. Codes: ***: 0.01, **: 0.05, *: 0.1 |  |

So whilst the relationship between the pre growth rate and the post growth rate is probably not zer ${ }^{11}$, the link is much less than one-to-one. We can also look at a plot of the two growth rates with the line of best fit overlaid on top. The gaps between the line of best fit and the observations are huge! It seems like past growth rates are not a great forecaster of future growth rates...


So this technique is far from ideal, but without going into more in-depth analysis, its hard to improve on this approach. So, for now, we'll have to make do with using past growth rates, though we should be aware that these are not great forecasters of future growth rates!

### 2.4.2 Using the growth rate approach

Now we have some structure on how to forecast future dividends, we can implement them to calculate the PV of a stock. Suppose we have found a measure for expected return by comparing the stock to other similar stocks and looking at their returns, or by using past returns of the same stock as a measure. Let's just call this $r$ to keep things simple. Suppose we've also found the past growth rate of the stock's

[^0]dividends, $g$, and we then apply the formula outline above:
$$
\mathbb{E}_{0}\left[D I V_{t}\right]=(1+g)^{t} D I V_{0}
$$

Then, note the following:

$$
\begin{aligned}
P_{0}=P V & =\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t} \mathbb{E}_{0}\left[D I V_{t}\right] \\
& =\sum_{t=1}^{\infty}\left(\frac{1}{1+r}\right)^{t}(1+g)^{t} D I V_{0} \\
& =D I V_{0} \sum_{t=1}^{\infty}\left(\frac{1+g}{1+r}\right)^{t} \\
& =D I V_{0} \frac{1+g}{1+r} \sum_{t=0}^{\infty}\left(\frac{1+g}{1+r}\right)^{t} \\
& =D I V_{0} \frac{1+g}{1+r} \frac{1}{1-\frac{1+g}{1+r}} \\
& =D I V_{0} \frac{1+g}{1+r} \frac{1}{\frac{1+r}{1+r}-\frac{1+g}{1+r}} \\
& =D I V_{0} \frac{1+g}{1+r} \frac{1}{\frac{r-g}{1+r}} \\
& =D I V_{0} \frac{1+g}{1+r} \frac{1+r}{r-g} \\
& =D I V_{0} \frac{1+g}{r-g}
\end{aligned}
$$

So, we have collapsed our expression down into something very simple! If I know what the dividends in the year that I bought the stock, and I have estimates of the growth rate and rate of return of the stock, then the PV, and hence the price, of the stock is given by:

$$
P_{0}=D I V_{0} \frac{1+g}{r-g}
$$

This expression is often simplified using the formula above:

$$
P_{0}=\frac{D I V_{1}}{r-g}
$$

## 3 Some finance 'jargon'

The key takeaway from this set of notes is that stock prices are functions of expected dividends. However, in this section, we'll just outline a couple of pieces of classic financial 'jargon' that typically accompany conversations about stocks so that you can feel very clever and important when you know what they mean!

### 3.1 Dividend Yield

The dividend yield is the annual dividend payments, divided by the price of the stock:

$$
\text { DivYield }_{1}=\frac{D I V_{1}}{P_{0}}
$$

Note that, if we assume a constant dividend growth rate, then we can rearrange the price formula above to be a function of the dividend yield and the expected growth rate of dividends:

$$
\begin{aligned}
P_{0} & =\frac{D I V_{1}}{r-g} \\
\therefore(r-g) & =\frac{D I V_{1}}{P_{0}} \\
\therefore r & =\text { DivYield }_{1}+g
\end{aligned}
$$

So the expected returns to a stock is the sum of the expected dividend yield, plus the expected growth rate of stocks. Recall our previous two examples of companies with low and high dividend yields respectively? In 2021, Amazon had a dividend yield of 0 , and Altria, the cigarette manufacturer, had a dividend yield of $8.57 \%$. Let's look at their three year average returns and three year average dividend yields and impute what the rate of $g$ would need to be to reconcile the two numbers:

|  | DivYield | Total Return | Implied $g$ |
| :---: | :---: | :---: | :---: |
| Amazon | 0 | $35.0 \%$ | $35.0 \%$ |
| Altria | $7.3 \%$ | $5.9 \%$ | $-1.4 \% \%$ |

This helps us understand that their are two types of stocks: growth stocks, like Amazon, and income stocks, like Altria. Given that cigarette use has been declining for some time, it's not surprising that the imputed growth rate for Altria is negative.

### 3.2 Earnings-price ratio

A very commonly reported financial indicator is Earnings-per-share, or EPS for short $\|^{2}$. The earnings-price ratio, $E P R$, is the ratio of $E P S$ to the price, $P_{0}$

$$
E P R=\frac{E P S}{P_{0}}
$$

Note that if a company pays out all their earnings as dividends, then:

$$
\begin{aligned}
E P S & =D I V \\
\therefore E P R & =\text { DivYield } \\
\therefore P_{0} & =E P R+g
\end{aligned}
$$

for a stock with an assumed constant growth rate of dividends. However, in any scenario in which the company does not pay out all earnings as dividends, then this will not hold.

Although earnings and dividends are not typically the same, there is clearly a link between these two objects. If a firm is not earning, it has no money to pay out in dividends. This is why earnings are often used as a proxy for dividends. Earnings has a number of statistical advantages over dividends:

- Earnings tend to be more stable
- Dividends can fluctuate quite wildly, so can offer a noisier impression of the underlying fundamentals of the firm
- Earnings are more predictable
- As well as being more stable, and thus more predictable, more attention is directed towards earnings by analysts than dividends.
- Many financial analysts work on forecasting earnings results for firms, providing additional information to the marketplace.

[^1]
[^0]:    ${ }^{1}$ Note the significance level!

[^1]:    ${ }^{2}$ By a happy coincidence, EPS are my full initials: (E)dward (P)eter (S)hore!

