Topic 9: Corporate Financing*

Eddie Shore
Corporate Finance – ECONS4280

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In this set of notes, we're going to start thinking about *corporate financing*. We've so far focussed on how to *value* projects. We now move to the question of *how to pay for them*. We start by outlining the difference between debt and equity, with a brief introduction to accounting principles. We will then discuss the WACC, or the *weighted average cost of capital*. We then move to an overview of some macro trends with respect to corporate financing. We then tackle the big question: how should companies finance themselves, with debt or equity? We will review the classic Modigliani-Miller Theorem, which tells us that it doesn't matter *at all*.

1 Debt vs. Equity

Throughout the course we've talked about different financial assets, be they bonds, stocks, or loans, and discussed their pricing and valuation. In the case of bonds, three things matter: the coupon payments, the principal, and the maturity. For stocks, we focused on risk and return, and the benefits to diversification. What we haven't discussed is how the *choice* between these two sources of finance affect the overall makeup of the firm. That's what we'll do now.

1.1 An intro to accounting

(*I know that 'an intro to accounting' does not sound like the most exciting topic imaginable, but a little bit of knowledge here is really important!*)

*Examples for Modigliani-Miller were taken from *excellent* notes from Jessica Wachter's Corporate Finance Handout notes. You can find them here

When a firm is deciding to raise cash, it can either do so by issuing debt, or issuing shares. The impact that this has on the balance sheet will depend on which channel is used.

The key principle that underpins modern accounting is something known as ‘double-entry’. The idea is that whenever ‘assets’ enter the company, they must be offset *exactly* by some liability *or* some change in the underlying *equity* of the company. For example, if I take out a loan to buy a new warehouse, the value of the warehouse enters my assets, *and* the value of the loan enters my liabilities! Alternatively, if I issue shares, then the *equity* portion of my balance sheet must increase by the *same amount* as the asset increase so as to equate both sides.

What does this look like in practice? To illustrate this with a figure, all balance sheets look something like the example below:

On the left side are the *assets*, and on the right side is the *sum* of the liabilities and the *equity*. So, when we buy that warehouse, we add the assets to the left, and then *either* add to liabilities (debt) or owner’s equity (shares) to *offset* the investment in the asset.

Typically, the assets of a company exceed their liabilities. This gap is the *net value* of the company. It thus makes sense that the *equity*, i.e. the value of *owning* the company, is captured by this difference.

2 The Weighted Average Cost of Capital

As we've just seen, a company's balance sheet tells us the distribution of assets and liabilities within the firm. On the left hand side is assets, and on the right hand side is liabilities. Typically the assets of a firm exceed its liabilities, and the size of this gap is simply the *equity* or the net assets or the net worth or capital of the company. According to the principles of accounting, net worth must equal assets minus liabilities. So, if we consider a simplified company balance sheet of the following form:

Assets	100	Debt	30
		Equity	70

then we can say that 30% of the company's value is made up of debt, and 70% of equity. Now suppose we want to talk about what the *cost* of raising capital for this firm would be. Well, if 30% of its funding has come from debt, and 70% from equity, and we imagine that the company continues to maintain this split, then a reasonable claim would be that the cost of raising capital is just the *weighted sum* of the cost of raising through debt *and* equity, where the weights are in proportion to this split! In mathematical form:

$$r = \frac{D}{D+E} r_D + \frac{E}{D+E} r_E$$

The expression above is known as the **weighted average cost of capital**, or *WACC*. The cost of raising money through debt is usually pretty clear: we are told by the bank what our interest rate is, for example. The cost of raising money through equity is a little more complicated. That's where our old friend, the CAPM comes in!

2.1 Using CAPM to establish the equity cost of capital

Unlike in the case of debt, it is not always immediately obvious what the cost of raising equity is. On the one hand, we can always create new shares 'costlessly' by just printing slips of paper. On the other hand, we now owe a debt of service to the new owners of our company: we need to deliver them dividends, growth, or both!

How much our new owners will expect to be rewarded for their investment is not always clear. Perhaps some new owners see this as a growth opportunity, whereas others are interested in cold hard cash! Either way, one rough metric we *can* use to uncover the *average* expectations of new shareholders is to find some way of establishing what the *expected* return of our company will be in the future. CAPM is one tool we can use to establish exactly this figure.

3 Macro Trends

So, how much do companies tend to use debt vs. equity? Consider the following graph:

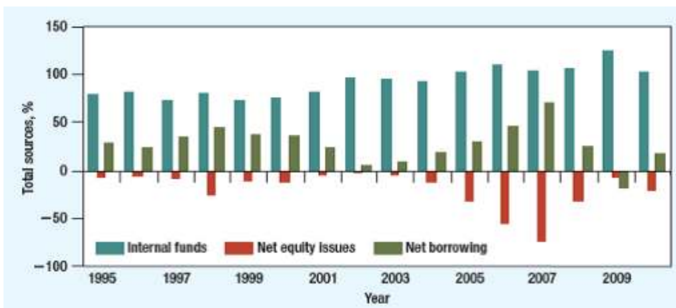


Figure 1: Source: BMA, 10th ed.

This graph shows us the sources of funds for U.S. non-financial firms expressed as a fraction of the total. Note that *internal financing* has been the modal choice across the entire period! So it seems like managers use *neither* equity, *nor* debt! Also, the net inflows from equity issues is *negative!* This means that companies *buy back* more shares than they issue.

Why might firms use internal funds? On the one hand, they need not pay interest on the use of these funds. On the other, they bear another cost by holding onto this stock of available cash: an opportunity cost! By holding onto cash, a firm is by definition *not investing that cash.*

3.1 Debt-Equity Ratios

So, what is the typical ratio between debt and equity for a US firm? Consider the following graph:

This graph shows the evolution of both the *market value* to debt ratio, and the *book value* to debt ratio for non-financial U.S. firms. In both cases, the value of debt has always been *less than 50%*, and usually around 40%.

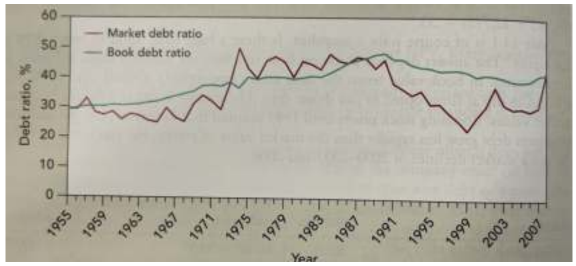


Figure 2: Source: BMA, 10th ed.

3.2 International Comparisons

How does this compare to other nations? Consider the following figure:

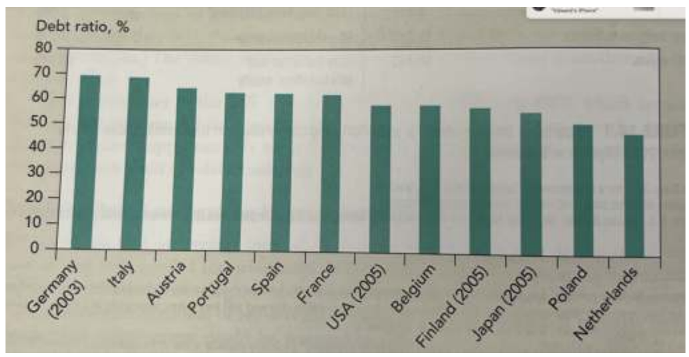


Figure 3: Source: BMA, 10th ed.

This figures shows the 'debt weight' that we'd calculate under the WACC, for manufacturing firms across 12 different countries. Germany has the highest, with the US being somewhere in the middle. *It is a classic description of German companies that they are funded by debt not equity.*

3.3 Who owns stocks?

Is it households? Governments? The following figure sheds some light:

The pie chart above shows the holders of corporate equities by category in the fourth quarter of 2008. For the most part, *households* make up the largest group of stock holders, closely followed by Pension funds and Mutual funds.

Sidenote: A mutual fund is an institution that pools investors money together and then manages those investments actively. A hedge fund is a *type* of mutual fund. The difference is that hedge funds impose *restrictions to entry* of investors,

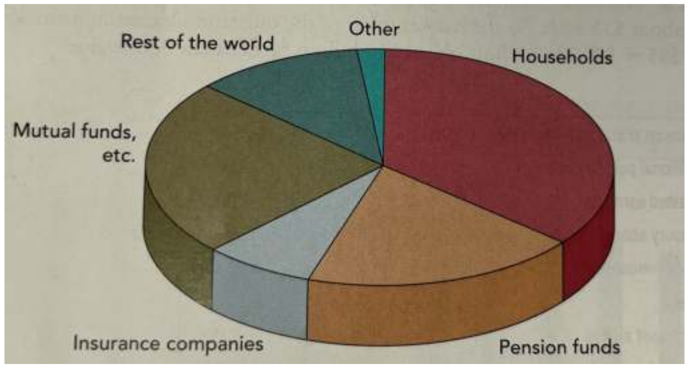


Figure 4: Source: BMA, 10th ed.

usually by way of a sizeable required investment, and hedge funds tend to go after *bigger, and hence riskier, returns.*

Before we get to rosy-eyed about the fact that households are the largest category of stockholder, take a look at the following graph from the Fed:

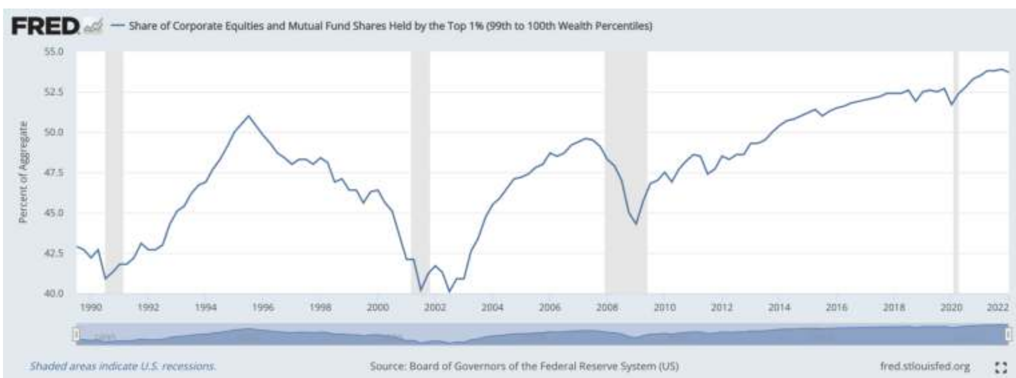


Figure 5: Source: FRED

This figure shows the share of corporate equities *and* mutual fund shares that are held by the top 1% of the population. A huge amount of these holdings are, therefore, concentrated amongst a very small number of the total population.

3.4 So, why don't more people own stocks?

A very good question! Here we move to the slides.

3.5 What about debt?

The following chart shows us the holdings of corporate bonds by 'category': Note

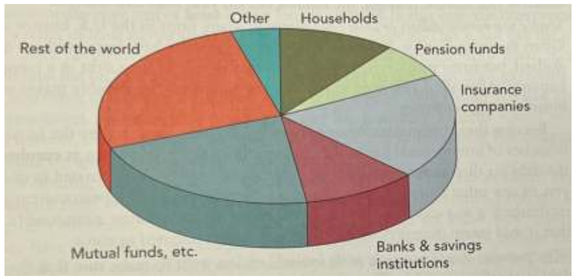


Figure 6: Source: BMA, 10th ed.

that households hold a much smaller proportion of corporate debt! There could be many reasons for this, but one likely explanation is that the barriers to entry in corporate bonds is much higher. This is a combination of institutional infrastructure (more difficult than trading stocks) and information frictions (less well advertised/salient than stocks).

4 Choosing the optimal 'capital structure' —Modigliani-Miller

The choice over whether to finance via debt or equity, commonly known as the choice over *capital structure*, is a classic question, though one that historically had a surprising answer. In 1958, two authors, Franco Modigliani[1] and Merton Miller established what has become known as the quintessential corporate finance theory: namely, that in *perfectly efficient capital markets,* capital structure *does not matter*. That is to say, a firm can freely choose between financing by debt *or* equity, *with no changes to the value of the firm*.

In this section we will explore that argument, identify its assumptions, and discuss its implications. Once we understand their argument, we will explore what happens when we relax some of the assumptions. What we will show is that capital structure *does* matter in practice, precisely because the assumptions are unlikely to bear out in the real world. Learning Modigliani-Miller is nonetheless a *hugely* valuable exercise, as it (i) teaches us to think carefully about firm value, and (ii)

[1]Fun fact: Modigliani's first academic job was at Columbia!

has influenced how we think about corporations, corporate structure, and indeed the efficiency of markets ever since.

4.1 Setting the scene

In this subsection, we consider the prospects of a manager seeking to maximise the value of their firm by adjusting their debt and equity levels. Suppose you are currently the manager of a firm that has zero debt. Let's suppose that you want to try to increase the value of your firm by issuing debt. How might that work?

Let's suppose that your earnings are random, but can only take one of three values depending on the state of the economy: $\{Recession, Normal, Boom\}$. In times of recession, you only earn \$100, in normal times you earn \$250, and during booms you manage to make \$300. Suppose that, every year, there is a 12.5% chance of a recession, a 50% chance of normal times, and a 37.5% chance of a boom. What are your expected earnings?

$$\begin{aligned}\mathbb{E}[Earnings] &= \frac{1}{8}100 + \frac{1}{2}250 + \frac{3}{8}300 \\ &= \$250\end{aligned}$$

As a firm with no debt, you are entirely funded by equity. Let's suppose you have 100 shares outstanding, and you pay all your earnings out in dividends. Note that this implies that expected earnings-per-share is equal to expected dividends-per-share, which is just $\frac{250}{100} = 2.5$. What is the price of your company's stock?

Recall that:

$$\begin{aligned}
P_t &= \mathbb{E}_t\left[\sum_{j=1}^{\infty}\left(\frac{1}{1+r}\right)^t DIV_{t+j}\right] \\
&= \sum_{j=1}^{\infty}\left(\frac{1}{1+r}\right)^t \mathbb{E}_t\left[DIV_{t+j}\right] \\
&= \sum_{j=1}^{\infty}\left(\frac{1}{1+r}\right)^t 2.5 \\
&= \frac{1}{1+r}2.5\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^t \\
&= \frac{1}{1+r}2.5\frac{1}{1-\frac{1}{1+r}} \\
&= \frac{1}{1+r}2.5\frac{1+r}{r} \\
&= \frac{1}{r}2.5
\end{aligned}$$

Suppose that you estimate the β of your firm, and use this to back out your expected return. You find that $r = 0.125$. Plugging this in:

$$\begin{aligned}
P_t &= \frac{1}{0.125}2.5 \\
&= \$20
\end{aligned}$$

Hence, your company's *value* is just $\$20 \times 100 = \$2,000$.

4.2 Can I improve value by issuing debt?

You now attempt to increase this value by taking on debt. Suppose you issue \$1,000 of debt, and use the proceeds to *buy back* shares. Using the \$1,000, you can buy 50 shares, and you keep the debt on your books forever. Suppose that the rate of return you have to pay for this debt is, let's say, 10%, meaning you are now obligated to pay \$100 in debt repayments *for good*. Whatever is left, you send out as dividends. What does this do to our expected payoffs?

So the question is: what has happened to the value of this company? Okay, here's where it gets a little nuanced. What we're going to do is *assume* that the

	Recession	Normal	Boom
Income	100	250	300
Interest payments	100	100	100
Dividends	0	150	200
DPS	0	3	4

company has become more valuable, and then show that this assumption will cause us to arrive at a *contradiction*. This is a very common way of proving things.

Okay, so let's suppose that the debt exercise above has *indeed* increased the value of the firm. Let's call this value, $V_H > \$2,000$. Note that there are now only 50 stocks remaining, meaning that each stock must be worth *more* than \$20, by definition. So buying a stock in this company is *more* expensive.

Does this make sense? Suppose we want back to the world before the debt issuance. Let's suppose I, as an outside investor, decide to buy *two* stocks in the unleveraged firm, but I have to borrow at the same rate as the company, 10%. The other one I just buy outright. What payoffs do I achieve?

	Recession	Normal	Boom
Dividends	2	5	6
Interest payments	2	2	2
Total	0	3	4

Well hang on! These payoffs are exactly the same as in the leveraged firm! And the stocks only cost me \$20, so the value of the firm has to be \$2,000: my decision to borrow to purchase the stocks has *nothing* to do with the fundamental properties of the firm *whatsoever*. So, before the firm issued debt, *I could already access these payoff structures* just by modifying my *own* debts. If the payoff structures are the same, *they must be equally valuable.* Thus it *cannot* be the case that *anyone* would value the new stocks at a price greater than \$20, *because the decision to increase the debt has created payoffs that were already possible.*

This result is known as **Proposition 1 of Modigliani-Miller**. Stated formally:

Definition (Proposition 1). The value of the firm is *independent* of the percentage of debt or equity in its capital structure.

4.2.1 Confused?

This is a little challenging to get your head around. The idea is that *changing the way that you chop up cash flows* should *not* generate wealth. Once we understand that our *own* investors have access to capital markets, *then there are many different cash flow structures that they can achieve without our manipulation.* As such, any manipulation that we make *does not change the options available to our owners.* If it does not change the options of cash flows available to our owners, why on earth would it increase the value of the company??

4.2.2 But the payoffs have changed! Why would the value be the same?

Note that once we take on debt, *we change the risk profile of our firm.* Note that our firm earnings are now *more* responsive to the state of the economy! So the *expected rate of return must* change. To find what the new rate must be, we can just plug in the formula. Note that now the expected dividend is \$3 in every period. So:

$$\begin{aligned}
P_t &= \frac{1}{r}\mathbb{E}_t[DIV] \\
\$20 &= \frac{1}{r}\$3 \\
\therefore r &= \frac{3}{20} \\
&= 0.15
\end{aligned}$$

So whilst capital structure *does* affect the returns of our stock, *it cannot change its overall value.* It does not change its *value*, because it *does not change which cash flows we can provide to shareholders.*

Is there a formal way of understanding exactly how much the additional leverage will increase the required rate of return on equity? Of course the answer is yes! Recall our definition of the WACC:

$$r = \frac{D}{D+E}r_D + \frac{E}{D+E}r_E$$

Note that previously, before leverage, the return was 12.5%. Now that the firm is leveraged, we know that the *value* of the firm is unchanged. As such, the WACC *must* be 12.5%, otherwise the weighted costs associated with owning the leveraged firm are *greater* than the unleveraged firm, which would necessarily imply a different valuation. With this in mind, let's perform some re-arrangement. Let r_0 denote the rate of return of the unleveraged firm, i.e. $r_0 = 12.5\%$. Then:

$$\begin{aligned}
r_0 &= \frac{D}{D+E} r_D + \frac{E}{D+E} r_E \\
\frac{D+E}{E} r_0 &= \frac{D}{E} r_D + r_E \\
\therefore r_E &= \frac{D+E}{E} r_0 - \frac{D}{E} r_D \\
&= r_0 + \frac{D}{E} r_0 - \frac{D}{E} r_D \\
&= r_0 + \frac{D}{E} (r_0 - r_D)
\end{aligned}$$

Let's check to see if this works out. Plugging in our numbers:

$$\begin{aligned}
r_E &= 0.125 + \frac{1000}{1000}(0.125 - 0.1) \\
&= 0.125 + 0.025 \\
&= 0.15
\end{aligned}$$

Exactly right! This relationship between the debt-equity ratio and the cost of equity capital forms the *second* proposition of Modigliani-Miller:

Definition (Proposition 2). The cost of equity capital is increasing in the percentage of debt in the capital structure. In fact,

$$r_E = r_0 + \frac{D}{E}(r_0 - r_D)$$

where r_0 is the cost of capital if the firm were financed entirely with equity.

4.3 How did we get there?

In this example, we were operating in a very simple world. Importantly, there was no tax. In the real world, governments tax companies on their *profits*. As debt reduces profits, *it also reduces a company's tax burden*. If we introduce taxes, we arrive at a very different conclusion, as we'll see in the next section.

Equally important, and really at the heart of our proof: shareholders had access to the *same interest rates* as us, the company. In this simple scenario, the decision over how to fund our activities *made no difference* to the cash flows available to our shareholders. There ability to borrow and save *at the same rate as us*, meant that *our* choice to modify debt really made no difference to the options available to shareholders. If shareholders wanted to back out the same payoffs as before we took on debt, *they could simply lend at the risk free rate*, and achieve the *exact same payoffs as before.*

Finally, we didn't really consider the possibility of bankruptcy. What if a firm cannot pay back its debts? In this scenario this wasn't possible, but even with a slight modification where recessions induce losses, we could still imagine some solution where we can engage in some bankruptcy arrangement that allows us to continue operating. In practice, bankruptcies are extremely costly, and create a non-trivial risk factor for over-leverage.

None of these assumptions are reasonable in real life. Nonetheless, the basic principles that *how you cut up cash flows doesn't matter*, and that shareholders *also* have access to capital markets are really critical to understand. As we now move on to the breakdown of MM, bear in mind why the modifications we make will mean that capital structure actually does matter.

5 Modifying Modigliani Miller, mmm!!

We'll start by introducing taxes. This is enough to break the MM finding, but creates its own problems. As we'll see, the presence of tax benefits from issuing debt implies that firms should *only* be funded by debt. This is not what we see in the real world, so is also a 'strange' conclusion to find. Once we also introduce the risks associated with *bankruptcy*, we will resolve the 100% debt recommendation, and arrive at a practical recommendation with respect to capital structure: namely, take on debt but not too much!

5.1 Adding taxes

Corporations are taxed on their profits. Here is a small overview of different tax rates across the globe:

If a firm makes less profits, *it pays less in tax.* It turns out that the introduction of this 'distortion' has a profound impact on the conclusions of Modigliani-Miller.

Country	Corporate Tax Rate
Albania	15%
China	25%
France	28.41 %
Iceland	20%
Israel	23%
South Korea	27.5%
United Kingdom	19%
United States of America	25.75%

Table 1: Source: Tax Foundation Report on Corporate Tax Rates around the World, 2021

We can illustrate that by extending our simple example.

5.1.1 Tax and our example case

Recall that in our example, by borrowing at the risk free rate, it was possible to create the following payoffs:

	Recession	Normal	Boom
Dividends	2	5	6
Interest payments	2	2	2
Total	0	3	4

Now suppose that *profits* are taxed at roughly the UK rate: 20%[2]. Note that there is *no tax shield in this example* from the debt that the *investor* has accrued. Then the payoffs to shareholders are as follows:

	Recession	Normal	Boom
Dividends	2	5	6
Interest payments	2	2	2
Tax	0.4	1	1.2 Total
-0.4	2	3	

Note this is per-share, so we can multiply everything by 100 to get the actual numbers of -$40, $200, and $300.

[2] Come on, not every example has to be the US! Indulge me!!

So, the average amount of *net* cash flows to the shareholders is $-\frac{1}{8}\$40 + \frac{1}{2}\$200 + \frac{3}{8}\$300 = \$207.50$. Now suppose we consider the same payoffs for the *leveraged* firm:

	Recession	Normal	Boom
Earnings	100	250	300
Interest payments	100	100	100
Tax	0	30	40
Dividends	0	120	160

Now the average amount paid out to the shareholders is $\frac{1}{8}0 + \frac{1}{2}120 + \frac{3}{8}160 = \$120$. **BUT** we are also paying out \$100 to debt holders *in every case*. So the amount of cash the firm pays out to *both* debtholders and shareholders is actually \$220. *This is more than the unleveraged firm!* The **tax shield** on profits has allowed the firm to disburse more wealth to its owners, *despite making the exact same earnings*: *both* debtholders *and* shareholders.

5.1.2 Formalising this notion

The introduction of tax creates a *shield* that allows the firm to pay out more to *both* shareholders and debtholders. We can formalise this with a simpler example that will be easier to follow.

Suppose there are two firms, identical in every way, except for capital structure—both make \$1,000 in income each year. One is unlevered, the other levered. The levered firm has borrowed \$4,000 to be paid back in equal installments *indefinitely*. To keep things simple, *suppose both firms deliver certain payoffs*, so their payoffs should be discounted by the *risk-free rate*. Let's suppose that rate is 10%.

How much are the interest payments? Well, if we borrowed at the risk-free rate of 10%, then the debt payments should be \$400 every year, forever. Let's put the cashflows of the two firms next to each others, and apply the tax argument from before:

So, the present value of the equity flows from the unlevered firm is given by the

	Unlevered	Levered
Operating Income	1,000	1,000
Interest	—	-400
Pre-tax Income	1,000	600
Tax at 20%	-200	-120
Equity income	800	480
Total cash flows to investors	800	880

following expression:

$$\begin{aligned} E^U &= \frac{800}{(1+0.1)} + \frac{800}{(1+0.1)^2} + ... \\ &= \sum_{j=1}^{\infty} \left(\frac{1}{1.1}\right)^t 800 \\ &= \frac{1}{r} 800 \qquad = \$8,000 \end{aligned}$$

The value of the equity flows from the *levered firm* is:

$$\begin{aligned} E^L &= \frac{460}{(1+0.1)} + \frac{460}{(1+0.1)^2} + ... \\ &= \sum_{j=1}^{\infty} \left(\frac{1}{1.1}\right)^4 60 \\ &= \frac{1}{r} 480 \\ &= \$4,800 \end{aligned}$$

And the value of the *debt* flows are:

$$\begin{aligned} D^L &= \frac{400}{(1+0.1)} + \frac{400}{(1+0.1)^2} + ... \\ &= \sum_{j=1}^{\infty} \left(\frac{1}{1.1}\right)^4 00 \\ &= \frac{1}{0.1} 400 \\ &= \$4,000 \end{aligned}$$

The total value of the levered firm is then the *sum* of these two flows, the flows to *all* investors in the firm. Note that the difference is just $\frac{1}{r}60$? *This is simply the present value of the tax shield*! We can show this by thinking about the total income that is shielded by tax each year: that is, the interest payments, multiplied by the tax rate.

$$
\begin{aligned}
PV(tax\ shield) &= \frac{0.2 \times 400}{(1+0.1)} + \frac{0.2 \times 400}{(1+0.1)^2} + ... \\
&= \sum_{j=1}^{\infty} \left(\frac{1}{1.1}\right)^t 80 \\
&= \frac{1}{1.1} 80 \\
&= \$800
\end{aligned}
$$

Generalising: let the tax rate be given by τ, and the total debt be D. It then follows that the interest payments, which create the tax shield, are just rD. Then, note that:

$$
\begin{aligned}
PV(tax\ shield) &= \frac{\tau \times rD}{(1+r)} + \frac{\tau \times rD}{(1+0.1)^2} + ... \\
&= \sum_{j=1}^{\infty} \left(\frac{1}{1+r}\right)^t (\tau \times rD) \\
&= \frac{1}{r}(\tau \times rD) \\
&= \tau D
\end{aligned}
$$

This brings us to the modified MM in the case of corporate taxes.

Definition (Proposition 1 with tax). The value of leveraged firms is:

$$V_L = V_U + \tau D$$

5.1.3 What about returns?

As before, we'll now think about what the tax increase implies for our rates of return. Note that Proposition 2 isn't going to work anymore: now that the value of the two firms is *different*, the WACC of both firms will *not* be the same.

What will the WACC look like once we introduce tax? Well, the *cost* of raising debt is now *diminished* by the tax rate. We only have to pay a *proportion* of the interest that we accrue, because some of it is *taken out of our tax bill.* The way we capture that is in the following way: let τ denote the corporate tax rate. Then:

$$r_{WACC} = \frac{D}{D+E}(1-\tau)r_D + \frac{E}{D+E}r_E$$

Here, we reduce the cost of debt to $(1-\tau)$ because the interest on the debt is *tax deductible.* The τr_D goes *unpaid for*, because we are not taxed on this amount of our debt. So, how do we go from this result to a solution for *our* WACC?

5.1.4 Returns: A worked example

Suppose we have a project that costs \$1m to put in place, and will generate \$95k *forever.* This project is *as risky* as the *typical* activities of the firm. What does that mean?? It means that the *firm-level* β is going to be a *good* way of constructing expected returns on *equity*, and the WACC will give us a measure of the expected return of the firm's activities *as a whole!*

Let's suppose that the firm's equity β is 0.75. Why have I said 'equity' β? Because we're only going to look at how the *equity* returns move with market returns, not the *total returns*, i.e. the equity *and* debt returns. Suppose that the risk free rate is 5%, the expected market return is 15%, and the company's cost of debt is *also* risk free, so $r_D = 5\%$.

Now, let's suppose that the firm is leveraged: 40% debt, and 60% equity. As in our previous examples, we imagine that this debt is *perpetual*: we never pay it down. Finally, suppose that the tax rate is 25%. Okay, that was a lot to remember! Let's put it all in a table, so we have a reference:

Parameter	Value
Cost	\$1,000,000
Per-period net cashflow	\$95,000
β	0.75
Expected Market Return, r_M	0.15
Risk free rate, r_{rf}	0.05
Tax rate, τ	0.25
Debt to Value, $\frac{D}{D+E}$	0.4

With all this, we can establish the WACC. Note that we know the cost of debt, it's just $r_D = 0.05$. All we need to find is r_E. Using the result from CAPM:

$$\begin{aligned}(r_E - r_{rf}) &= \beta(r_M - r_{rf}) \\ \therefore r_E &= r_{rf} + \beta(r_M - r_{rf}) \\ &= 0.05 + 0.75 \times (0.15 - 0.05) \\ &= 0.125\end{aligned}$$

So, applying this to the WACC formula:

$$\begin{aligned}r_{WACC} &= \frac{D}{D+E}(1-\tau)r_D + \frac{E}{D+E}r_E \\ &= 0.4 \times (1 - 0.25) \times 0.05 + 0.6 \times 0.125 \\ &= 0.09\end{aligned}$$

So the average cost of capital to the firm is 9%. Note that this will *not* be the same as the pure equity firm!

5.1.5 An application to Present Value

If the *value* of the firm has changed, and so the WACC has changed. We can use the WACC as our measure of the *cost of capital* of the firm.

Now we can use this formula and *apply it to the Present Value formula we've* *used before*. Let x_t denote the *net* flows from this project.

$$
\begin{aligned}
PV &= \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \mathbb{E}[x_t] \\
&= -1,000,000 + \frac{1}{1+0.09}95,000 + \left(\frac{1}{1+0.09}\right)^2 + ... \\
&= -1,000,000 + \sum_{t=1}^{\infty}\left(\frac{1}{1+0.09}\right)^t 95,000 \\
&= -1,000,000 + 95,000\frac{1}{1+0.09}\sum_{t=0}^{\infty}\left(\frac{1}{1+0.09}\right)^t \\
&= -1,000,000 + 95,000\frac{1}{1+0.09}\frac{1}{1-\frac{1}{1+0.09}} \\
&= -1,000,000 + 95,000\frac{1}{1+0.09}\frac{1+0.09}{0.09} \\
&= -1,000,000 + \frac{95,000}{0.09} \\
&= \$55,555
\end{aligned}
$$

Okay, good news!! It's a good project :)

5.1.6 Is the PV the same if we are unlevered?

It shouldn't be, right? The WACC is different for the two firms! Note that we can uncover a novel equation that modifies Proposition 2 to allow for debt. Let r_0 be the WACC of the unlevered firm. It looks something like this:

$$r_0 = \frac{E}{(1-\tau)D+E}r_E + \frac{(1-\tau)D}{(1-\tau)D+E}r_D$$

It turns out there's an even more insightful representation of proposition 2 when we introduce tax. It can be shown that:

$$\beta_0 = \frac{E}{(1-\tau)D+E}\beta_E + \frac{(1-\tau)D}{(1-\tau)D+E}\beta_D$$

So the β of the unlevered firm is just a weighted combination of the β_E and β_D of the levered firm. So, let's apply these formulae, and uncover the PV of our project *were we to be unlevered*. Note that $\beta_D = 0$, because we assumed that we can borrow at the *risk-free rate*. Then:

$$
\begin{aligned}
\beta_0 &= \frac{E}{(1-\tau)D+E}\beta_E \\
&= \frac{1}{(1-\tau)\frac{D}{E}+1}\beta_E \\
&= \frac{1}{(1-0.25)\times\frac{0.4}{0.6}+1}\times 0.75 \\
&= 0.5
\end{aligned}
$$

Thus, under CAPM, it follows that:

$$
\begin{aligned}
r_0 &= r_{rf} + \beta_0(r_M - r_{rf}) \\
&= 0.05 + 0.5\times(0.15-0.05) \\
&= 0.1
\end{aligned}
$$

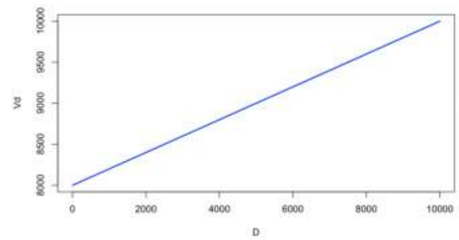
So, what is the NPV?

$$
\begin{aligned}
PV &= \sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t \mathbb{E}[x_t] \\
&= -1,000,000 + \frac{1}{1+0.1}95,000 + \left(\frac{1}{1+0.1}\right)^2 + ... \\
&= -1,000,000 + \sum_{t=1}^{\infty}\left(\frac{1}{1+0.1}\right)^t 95,000 \\
&= -1,000,000 + 95,000\frac{1}{1+0.1}\sum_{t=0}^{\infty}\left(\frac{1}{1+0.1}\right)^t \\
&= -1,000,000 + 95,000\frac{1}{1+0.1}\frac{1}{1-\frac{1}{1+0.1}} \\
&= -1,000,000 + 95,000\frac{1}{1+0.1}\frac{1+0.1}{0.1} \\
&= -1,000,000 + \frac{95,000}{0.1} \\
&= -\$50,000
\end{aligned}
$$

This is huge! The capital structure not only influences the value of the firm, *but also what projects* <u>*have*</u> *value for the firm.*

5.1.7 But we can't rack up debt forever, right?

One thing we have implicitly assumed throughout is that, as we raise the amount of debt, the rate of return that we pay is *independent* of the amount of debt that we have. If we were to plot the value of the firm as a function of the debt, then using Proposition 1 of MM with taxes, which states that $V_L = V_U + \tau D$ we would have something like the following:



Here I've used the values from the example above. Note that the maximum debt we could select is \$10,000, as at this point, the debt repayments would exceed the interest, which we will rule out.

Of course in reality, the more debt we have, the riskier we are, and the greater our cost of raising more debt. We can incorporate this idea by thinking about our rate of raising debt as being an *increasing* function in the interest rate r. Suppose that:

$$r_D = r_{rf} + \frac{1}{\alpha} D^{\psi}$$

where α and ψ are parameters that measures how quickly the increase in debt increases our interest rate. We can show how the rate on debt would increase, letting $\alpha = \psi$ as we increase debt for several different values of ψ by plotting them together:

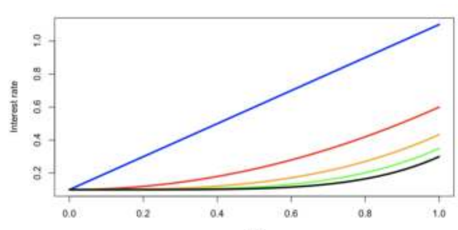
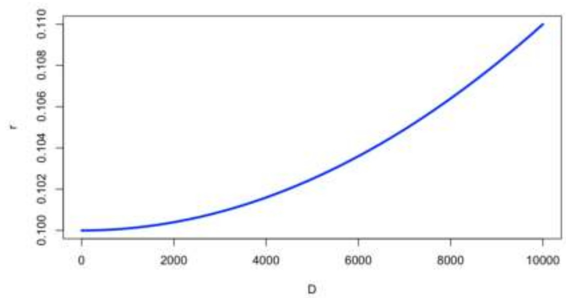
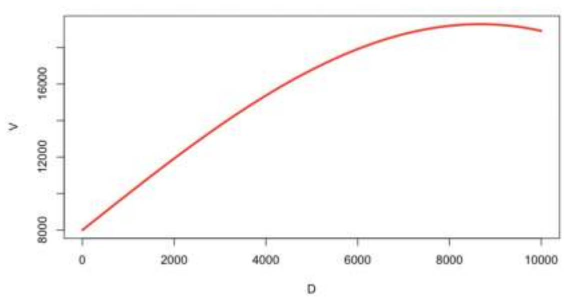


Figure 7: How the interest rate increases with debt for different value of ψ. The top line is $\psi = 1$. Then $\psi = 2$, $\psi = 3$, and so on.

In this scenario, what would our Proposition 1 look like? The mathematics behind this is a little intense, but suppose we selecting α and ψ so that the relationship between debt and the interest rate looked something like this:



So, if we max out our debt, the interest rate goes up to 11%. What does this mean for firm value? Suppose I now plot the value of the firm as the debt increases. Whilst my *tax shield* grows, *the cost of raising debt also grows.* What you find is something like the following:



So, once we take into account that increasing the debt *also* increases the cost of raising debt, *there is a point past which further debt lowers* the value of the firm. There is a *sweet spot*, around $8,200, after which the value of our firm *goes down.*

5.2 Bankruptcy

Note that in our models so far, we've always assumed that, even in bad states, the company makes enough money to pay its debts. What about when we relax that idea? One way of modelling this is to say that whenever a company can't pay its debts, it simply announces bankruptcy, and is allowed to continue operations at no cost. This is the way that MM model bankruptcy in their model.

However, in the real world, there are *huge* costs associated with going bankrupt. Some of these are:

- Legal costs of restructuring
- Loss of customers and suppliers
- Loss of employees
- Loss of receivables

So imagining that when a firm doesn't pay its debts it can simply walk away without cost is highly unrealistic.

5.2.1 How do we introduce bankruptcy costs?

Bankruptcy costs only occur when we cannot pay back our debts. This happens when we have a *bad shock*. I don't think it is worth our while working through an actual mathematical model here, because it will get too complicated. However, we will go through the basic idea.

Suppose that there are n different random states that could occur, each of which carries a different earnings profile for the firm. Let all the states be given by $\{z_1, z_2, ..., z_{n-1}, z_n\}$. Let the earnings in each state be given by $\{e_1, e_2, ..., e_{n-1}, e_n\}$. Let the probabilities of each state, $\{p_1, p_2, ..., p_{n-1}, p_n\}$ be such that $p_1+p_2+...+p_n = 1$. Suppose a firm has an amount of debt, D, and has to pay interest on that debt, r_D. For now, suppose that the interest rate is *fixed*, so we ignore the previous discussion. Finally, assume that corporate profits are taxed at rate τ. What does all of this look like?

	z_1	z_2	...	z_{n-1}	z_n
Earnings	e_1	e_2	...	e_{n-1}	e_n
Interest	$r_D \times D$	$r_D \times D$	...	$r_D \times D$	$r_D \times D$
Tax	$(e_1 - r_D D)\,\tau$	$(e_2 - r_D D)\,\tau$	...	$(e_{n-1} - r_D D)\,\tau$	$(e_n - r_D D)\,\tau$
Profit	$(e_1 - r_D D)\,(1-\tau)$	$(e_2 - r_D D)\,(1-\tau)$	...	$(e_{n-1} - r_D D)\,(1-\tau)$	$(e_n - r_D D)\,(1-\tau)$

Now suppose that in the really bad states, we run the possibility of incurring bankruptcy. In these scenarios we have to pay a cost for going bankrupt—our profits this period are *even lower* because we failed to meet our debt obligations. All this does is *lower the expected payoffs of the firm*.

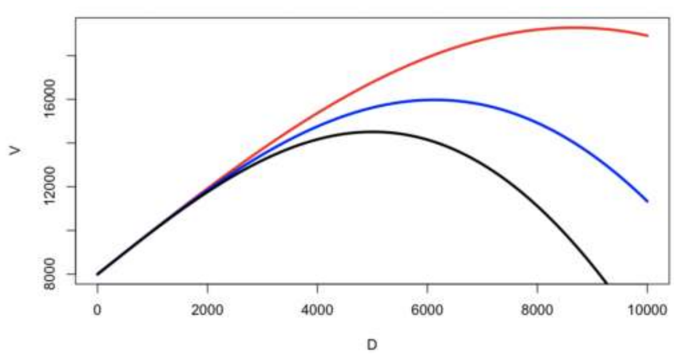
Note that the more debt we take on, *the more likely it is that we will fail to pay back debt*, as more states lead to scenarios in which our earnings fall to meet our debt obligations. So as debt rises, the expected payoffs of the firm *fall*, because we have more states in which we take on the *bankruptcy* cost.

What does this look like in practice? Suppose that once again we assume that the interest rate on debt increases as our debt increases (this seems very sensible in this case given our previous argument). Suppose we look at three firms: the first is our risk-free firm above. This firm will never go bankrupt, because it will never be allowed to borrow in such a way that it fails to pay its debts: because its earnings are always the same, *it would then fail to pay its debts every period*.

The second is a firm that, for a given value of D, has some small risk of failing to pay its debts. This risk is increasing in D, but suppose that even for $D = \$10,000$, there are only a few cases in which the firm cannot pay off their interest. When this happens, the firm takes on an *extra* cost. Nonetheless, as the firm takes on more debt, it will increase the number of states in which it cannot pay the interest payments, and so the *expected* cash flows of the company must drop, at least with respect to bankruptcy costs.

Finally, consider a third firm that, for the same value of D, has many more states of the world in which it fails to meet interest payments. This firm will suffer greatly as it increases its debt: the impact on its expected cash flows is sizeable because there is a good chance they will end up in a state where they take on bankruptcy charges.

What does this look like? I have done it for you! The three cases can be summed up in the following figure:



The top is the risk-free firm, repeated. Then the second firm is in blue, and the third firm in black. The finding is clear: as the risk of engaging in bankruptcy proceedings increases, *the optimal level of debt falls*. Cool finding huh?

5.2.2 How good is the theory?

Is it true? Do riskier firms have *lower* debt-to-equity ratios? This is something we can test! Note that if firm is less risky, it should have a low β. So, does a firm's β correspond with its debt to equity ratio? Let's check!

Dependent Variable:	$\frac{D}{E}$
Model:	(1)
Variables	
(Intercept)	0.1903***
	(0.0039)
β	-0.0069***
	(0.0024)
Fit statistics	
Observations	7,867
R^2	0.00104

IID standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Would you look at that!!! Not too shabby!!